The thermomagnetic effect in a semiconductor superlattice in the presence of laser radiation

1993 J. Phys.: Condens. Matter 5 1349

(http://iopscience.iop.org/0953-8984/5/9/019)

J. Phys.: Condens. Matter 5 (1993) 1349–1354. Printed in the UK

The thermomagnetic effect in a semiconductor superlattice in the presence of laser radiation

S Y Mensah and P K Buah-Bassuah
Department of Physics, University of Cape Coast, Cape Coast, Ghana

Received 28 April 1992, in final form 23 October 1992

Abstract. A theoretical investigation of the photostimulated thermomagnetic effect by electrons in a semiconductor superlattice (SL) in the lowest miniband is presented. The possibility of controlling the thermopower α, the electron thermal conductivity χ and the electroconductivity σ of the SL with the help of laser radiation is indicated. The parameters α, χ and σ were found to oscillate. The Hall constant, the Righi–Leduc and the Maggie–Righi–Leduc effects were calculated. The prospect of using the SL as a good-quality and highly efficient thermoelement is also proposed.

1. Introduction

In recent years, it has become difficult to find any field of science where attempts are not being made to determine new phenomena. The advent of the laser has rendered these searches practicable. It is known that strong laser radiation can influence the kinetic phenomena in semiconductors by changing not only the concentration of carrier charges but also the scattering probability. It is also interesting to note that the presence of laser radiation can cause many effects in semiconductors, e.g. odd magnetoresistivity, longitudinal magnetoresistivity, transverse radio and acousto-electronic effects. Different mechanisms are responsible for these anomalies, but one mechanism which is well studied is the situation when laser radiation affects the scattering probability of conducting electrons with phonons and impurities.

This mechanism is possible when the photon energy $\hbar\omega$ is comparable with or greater than the average energy $\bar{\epsilon}$ of the carrier charge. In contrast, when $\hbar\omega \ll \bar{\epsilon}$, a different non-linear mechanism comes into play. One such non-linearity is the case when the energy band of the semiconductor is non-parabolic (the dependence of electron velocity on its quasi-impulse is non-linear). The above-mentioned mechanism is more favourable in a semiconductor with thin forbidden and permissible bands which is known as a superlattice (SL).

The photostimulated thermomagnetic effect in homogeneous semiconductors has been studied [1,2]. In [3], it is indicated that, to study the photostimulated kinetic effect in homogeneous materials, there is the need to consider the quadrupole deformation of the distribution function. In [4], it is proved that, in the presence of laser radiation, the thermomagnetic effect can exist in a fully degenerate semiconductor.

In the present paper, the photostimulated thermomagnetic effect in a semiconductor SL is considered. It is necessary to note that this allows the possibility

0953-8984/93/091349+06$07.50

of controlling the thermomagnetic properties of the SL in a similar way to the case of the static conductivity of an SL [5].

2. Theoretical analysis

To solve the problem under consideration, the non-quantized magnetic field $\boldsymbol{H}$ is directed along the OZ axis and is perpendicular to the SL axis (OX axis). The constant field $\boldsymbol{E}_0$ and the incident laser radiation $\boldsymbol{E}(t) = \boldsymbol{E}_1 \cos(\omega t)$ lie in the XOY plane. $\boldsymbol{E}_0$ and $\boldsymbol{E}(t)$ are not necessarily collinear but are perpendicular to $\boldsymbol{H}$. $\boldsymbol{E}(t)$ makes an arbitrary angle ϕ with the OX axis. Furthermore, we shall consider a quasi-classical case, i.e. 2Δ is much larger than any other characteristic energy ($\hbar\omega$, $\hbar/\tau$, eE_0d, $dk_B\nabla_x T$ or eE_1d) where ω is the frequency of the high field, 2Δ is the width of the miniband of the SL, d is the period of the SL, τ is the relaxation energy of electrons, k_B is Boltzmann's constant, e is the electron charge, $\hbar$ is Planck's constant divided by 2π, and ∇T is the temperature gradient. These conditions are necessary to confine the electrons in the lowest miniband and also to allow the use of the kinetic Boltzmann equation. The collision integral is taken in the τ approximation, and further τ is assumed to be a constant [6,7].

The distribution function $f(\boldsymbol{p}, \boldsymbol{r}, t)$ of electrons in the lowest miniband of the SL is found by solving the kinetic equation

$$\begin{aligned}\partial f(\boldsymbol{p},\boldsymbol{r},t)/\partial t &+ [\partial f(\boldsymbol{p},\boldsymbol{r},t)/\partial \boldsymbol{r}]\boldsymbol{V}(\boldsymbol{p}) \\ &+ e\{\boldsymbol{E}(t) + \boldsymbol{E}_0 + (1/c)[\boldsymbol{V}(\boldsymbol{p}), \boldsymbol{H}]\}[\partial f(\boldsymbol{p},\boldsymbol{r},t)/\partial \boldsymbol{p}] \\ &= -(1/\tau)[f(\boldsymbol{p},\boldsymbol{r},t) - f_0(\boldsymbol{p})]. \end{aligned} \tag{1}$$

Here $\boldsymbol{V}$ and $\boldsymbol{p}$ are the velocity and momentum, respectively, of the electrons. Equation (1) is solved in a linear approximation of $\boldsymbol{E}_0$, ∇T and $\boldsymbol{H}$ using the iteration method and the initial conditions $f(\boldsymbol{p}, -\infty) = f_0(\boldsymbol{p})$.

The current density is defined as

$$j(t) = e\sum_{\boldsymbol{p}} \boldsymbol{V}(\boldsymbol{p}) f(\boldsymbol{p}, \boldsymbol{r}, t) \tag{2}$$

and the density of the heat current is expressed as

$$q(t) = \sum_{\boldsymbol{p}} [\epsilon(\boldsymbol{p}) - \xi] \boldsymbol{V}(\boldsymbol{p}) f(\boldsymbol{p}, \boldsymbol{r}, t). \tag{3}$$

After inserting the solutions of equation (1) into equations (2) and (3) and averaging over the period of incident radiation, we obtain the current density

$$\begin{aligned} j_i = -\frac{e\omega}{2\pi\tau} \int_0^{2\pi/\omega} \mathrm{d}t \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t_1} \mathrm{d}t_2 \exp\left(-\frac{t-t_2}{\tau}\right) \\ \times \sum_{\boldsymbol{p}} \left((eE_k - \nabla_k \xi) + [\epsilon(\boldsymbol{p}) - \xi] \frac{\nabla_k T}{T} \right) \\ \times V_i \left(\boldsymbol{p} + e\int_{t_2}^{t} \boldsymbol{E}(t')\,\mathrm{d}t' \right) \frac{\partial f_0(\boldsymbol{p})}{\partial p_k} \end{aligned}$$

$$+\frac{e^2\omega}{2\pi\tau c}\int_0^{2\pi/\omega} \mathrm{d}t \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t_1} \mathrm{d}t_2 \int_{-\infty}^{t_2} \mathrm{d}t_3 \exp\left(-\frac{t-t_3}{\tau}\right)$$
$$\times \sum_{p}\left((eE_k - \nabla_k\xi) + [\epsilon(p)-\xi]\frac{\nabla_k T}{T}\right) V_i\left(p + e\int_{t_3}^{t} E(t')\,\mathrm{d}t'\right)$$
$$\times \left[V\left(p + e\int_{t_3}^{t} E(t')\,\mathrm{d}t'\right), H\right]_j \frac{\partial^2 f_0(p)}{\partial p_k\,\partial p_j}. \tag{4}$$

Here i, j and k are Cartesian indices. The summation over repeated indices is assumed.

Analogously, the thermal-current density is expressed as

$$q_i = \frac{\omega}{2\pi\tau}\int_0^{2\pi/\omega} \mathrm{d}t \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t_1} \mathrm{d}t_2 \exp\left(-\frac{t-t_2}{\tau}\right)$$
$$\times \sum_{p}[\epsilon(p)-\xi]\left((eE_k - \nabla_k\xi) + [\epsilon(p)-\xi]\frac{\nabla_k T}{T}\right)$$
$$\times V_i\left(p + e\int_{t_2}^{t} E(t')\,\mathrm{d}t'\right)\frac{\partial f_0(p)}{\partial p_k} + \frac{e\omega}{2\tau c\pi}$$
$$\times \int_0^{2\pi/\omega} \mathrm{d}t \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t_1} \mathrm{d}t_2 \int_{-\infty}^{t_2} \mathrm{d}t_3 \exp\left(-\frac{t-t_3}{\tau}\right)$$
$$\times \sum_{p}[\epsilon(p)-\xi]\left((eE_k - \nabla_k\xi) + [\epsilon(p)-\xi]\frac{\nabla_k T}{T}\right)$$
$$\times V_i\left(p + e\int_{t_3}^{t} E(t')\,\mathrm{d}t'\right)\left[V\left(p + e\int_{t_3}^{t} E(t')\,\mathrm{d}t'\right), H\right]_j \frac{\partial^2 f_0(p)}{\partial p_k\,\partial p_j}. \tag{5}$$

The energy of the electrons in the lowest miniband is

$$\epsilon(p) = (P_y^2 + P_z^2)/2m + \Delta\cos(P_x d/\hbar) \tag{6a}$$

and the velocity of the electrons is expressed as

$$V(p) = [(\Delta d/\hbar)\sin(p_x d/\hbar),\ P_y/m,\ P_z/m]. \tag{6b}$$

We substitute equation (6*a*) and equation (6*b*) in equation (4) and equation (5) and solve the resulting equations for j_i and q_i, for a non-degenerate electron gas. The results are expressed as follows:

$$j_i = \sigma_{ik}E_k^* - \beta_{ik}\nabla_k T \tag{7}$$

$$q_i = \gamma_{ik}E_k^* - \chi_{ik}\nabla_k T \tag{8}$$

where $E^* = E_0 - \nabla\xi$. Since the representation of q_i in terms of E^* is not convenient when comparing theory with experiment, it becomes necessary to express q in terms of j and ∇T. Hence,

$$q_i = \alpha_{ik}Tj_k - \lambda_{ik}\nabla_k T. \tag{9}$$

The tensor components in equation (7) and equation (9) are σ_{ik} (electroconductivity), α_{ik} (thermopower), λ_{ik} (thermal conductivity), and β_{ik} and are given by

$$\begin{aligned}
\sigma_{xx} &= \sigma_{\|}[J_0^2(z\cos\phi) + \tfrac{1}{2}\Omega z\tau \sin\phi\, J_1(z\cos\phi)] \\
\sigma_{yx} &= -\sigma_{xy} = \sigma_{\|}\Omega\tau J_0^2(z\cos\phi) \qquad (10)\\
\sigma_{yy} &= ne^2\tau/m = \sigma_{\perp}
\end{aligned}$$

$$\begin{aligned}
\beta_{xx} &= -(k_B/e)\sigma_{xx}\{\Delta^* - \xi^* - 3 - \Delta^*[I_0(\Delta^*)/I_1(\Delta^*)]\} \\
\beta_{xy} &= -\beta_{yx} = (k_B/e)\sigma_{xy}\{\Delta^* - \xi^* - 3 - \Delta^*[I_0(\Delta^*)/I_1(\Delta^*)]\} \qquad (11)\\
\beta_{yy} &= -(k_B/e)\sigma_{yy}\{\Delta^* - \xi^* + 2 - \Delta^*[I_1(\Delta^*)/I_0(\Delta^*)]\}
\end{aligned}$$

$$\begin{aligned}
\alpha_{xx} &= -(k_B/e)[\![\Delta^* - \xi^* + 1 + \{2 - \Delta^*[I_0(\Delta^*)/I_1(\Delta^*)]\}\vartheta_2]\!] \\
\alpha_{xy} &= -\alpha_{yx} = -(k_B/e)[\![\Delta^* - \xi^* + 1 + \{2 - \Delta^*[I_0(\Delta^*)/I_1(\Delta^*)]\}\vartheta_1]\!] \qquad (12)\\
\alpha_{yy} &= -(k_B/e)\{2 + (3/2m)(eE_1\tau)^2/k_BT + \Delta^* - \xi^* - \Delta^*[I_1(\Delta^*)/I_0(\Delta^*)]\}
\end{aligned}$$

$$\begin{aligned}
\lambda_{xx} &= (\sigma_{xx}k_B^2T/e^2)[\![1 - \{2 + \Delta^*[I(\Delta^*)/I_1(\Delta^*)] \\
&\qquad + (\Delta^*)^2 - (\Delta^*)^2[I_0^2(\Delta^*)/I_1^2(\Delta^*)]\}\vartheta_2]\!] \\
\lambda_{xy} &= -\lambda_{yx} = (\sigma_{xx}k_B^2T/e^2)[\![1 + \{2 + \Delta^*[I_0(\Delta^*)/I_1(\Delta^*)] \\
&\qquad + (\Delta^*)^2 - (\Delta^*)^2[I_0^2(\Delta^*)/I_1^2(\Delta^*)]\}\vartheta_1]\!] \qquad (13)\\
\lambda_{yy} &= (\sigma_{\perp}k_B^2T/e^2)\{2 - \Delta^*[I_1(\Delta^*)/I_0(\Delta^*)] + (\Delta^*)^2 - (\Delta^*)^2[I_1^2(\Delta^*)/I_0^2(\Delta^*)]\}
\end{aligned}$$

where $z = eE_1d/\hbar\omega$, ϕ is the angle between $E(t)$ and the SL axis,

$$\begin{aligned}
&\Delta^* = \Delta/k_BT \qquad \sigma_{\|} = e^2nd^2\tau\Delta I_1(\Delta^*)/\hbar I_0(\Delta^*) \qquad \xi^* = \xi/k_BT \\
&\Omega = eH/mc \qquad E^* = E_0 - \nabla_z\xi \qquad \vartheta_1 = J_0^2(2z\cos\phi)/J_0^2(z\cos\phi) \\
&\vartheta_2 = [J_0^2(2z\cos\phi) + \tfrac{1}{2}\Omega\tau z\sin\phi\, J_1(2z\cos\phi)]/J_0^2(z\cos\phi) + \tfrac{1}{2}\Omega\tau z\sin\phi\, J_1(z\cos\phi)
\end{aligned}$$

J_0 and J_1 are Bessel functions, and I_0 and I_1 are modified Bessel functions.

Using the calculated tensors, it is possible to study different thermomagnetic effects as follows.

First the Hall constant is defined as

$$\begin{aligned}
R &= (1/H)(\sigma_{xy}/\sigma_{xx}\sigma_{yy}) \\
&= J_0^2(z\cos\phi)/enc[J_0^2(z\cos\phi) + \tfrac{1}{2}\Omega\tau z\sin\phi\, J_1(z\cos\phi)]. \qquad (14)
\end{aligned}$$

Secondly the thermal conductivity of an electron gas in a transverse magnetic field is defined as

$$\chi(H) = q_x/\nabla_x T$$

where $j_x = j_y = 0$ and $\nabla_y T = 0$. Then

$$\chi(\boldsymbol{H}) = (\sigma_{xx} k_B^2 T / e^2) [\![1 + \{2 + \Delta^* [I_0(\Delta^*)/I_1(\Delta^*)] + (\Delta^*)^2 - (\Delta^*)^2 [I_0^2(\Delta^*)/I_1^2(\Delta^*)]\} \vartheta_2]\!] . \tag{15}$$

The difference $\chi(H) - \chi(0)$ defines the Maggie–Righi–Leduc effect.

Thirdly the Righi–Leduc effect is expressed as

$$S = (\mu \sigma_{xy} k_B^2 T / \chi_L c e^2) [\![1 + \{2 + \Delta^* [I_0(\Delta^*)/I_1(\Delta^*)] + (\Delta^*)^2 - (\Delta^*)^2 [I_0^2(\Delta^*)/I_1^2(\Delta^*)]\} \vartheta_1]\!] \tag{16}$$

where χ_L is the lattice thermal conductivity and μ is the electron mobility.

3. Results, discussion and conclusion

We have obtained analytic expressions for the electron conductivity, the thermopower, the electron thermal conductivity, the Hall constant, the Righi–Leduc effect, etc. It is observed that the presence of laser radiation makes the above-mentioned effects oscillate. This oscillation can be associated with the fact that the change in quasi-momentum in the presence of the incident $\Delta p \simeq eE_1/\omega$ can be compared with the size of the Brillouin miniband (about $\hbar/d$) and thus the dependence of the velocity on the incident radiation assumes an oscillatory character.

It is worth noting that in the absence of the incident radiation our results assume the usual form, e.g. the Hall constant becomes $R = (enc)^{-1}$.

As indicated in [8], in the presence of laser radiation the Onsager relations are not satisfied. The diagonal component of the effective conductivity tensor σ_{xx} in the direction of the SL axis contains a term linear in the magnetic field. Thus the magnetoresistivity $\rho_{xx} \simeq \sigma_{xx}^{-1}$ becomes odd and hence is referred to as the odd magnetoresistivity.

Another interesting result can be obtained when $J_0(a\cos\vartheta) \simeq 0$ ($a\cos\vartheta \simeq 2.40, 2.52, \ldots$). In the absence of a magnetic field, σ_{xx} becomes zero, which implies that there are no current drifts in the direction of the SL axis, giving rise to a phenomenon referred to as the total self-induced transparency. However, the moment that the magnetic field is switched on, the current reappears.

This photomagnetostimulated conductivity can be either positive or negative (i.e. the current will drift in the direction of the field or opposite to it) depending on the sign of the magnetic field and on the magnitude and direction of the laser radiation.

For $\Delta \ll k_0 T$ and using the well known expressions for I_0 and I_1 at small values of the argument [9], in the absence of laser radiation, the thermopower α_{ik} and thermal conductivity λ_{ik} become

$$\alpha_{xx} = (k/e)(-\xi^* + 1) \tag{17}$$

$$\lambda_{xx} = k_B^2 T(\sigma_\parallel / e^2) \tag{18}$$

$$\alpha_{yy} = (k/e)(-\xi^* + 2) \tag{19}$$

$$\lambda_{yy} = k_B^2 T(2\sigma_\perp / e^2) \tag{20}$$

where

$$\xi = k_B T \ln(\pi\hbar^2 n_e d/mk_0 T). \tag{21}$$

Equation (17), which is the thermopower along the SL axis, was first obtained by Shik [10]. Equation (19), which is the expression for a quasi-two-dimensional semiconductor QWS, was obtained by Kubakaddi *et al* [11]. In [11], it was stated that the thermopower is enhanced above its value in the homogeneous (bulk) semiconductor for lower values of d.

In this theoretical investigation it has been observed that the presence of laser radiation enhances the thermomagnetic effects in an SL by the factors ϑ_1 and ϑ_2. These factors are amplitude dependent. Therefore, by changing the amplitude of the laser, the parameters and the effects mentioned earlier also change. In our opinion, an optimal selection of Δ and d for the SL can allow the use of an SL in the presence of laser radiation as a good-quality efficient thermoelement [7].

References

[1] Jackcimov E K 1981 *Fiz. Tekh. Poluprov.* **15** 407

[2] Malevich V L and Epshstein E M 1976 *Izv. Vyssh. Uch. Zaved. Fiz.* **19** 121

[3] Nguyen Xonh Shon and Shmelev G M 1985 *Fiz. Tekh. Poluprov.* **19** 58

[4] Nguyen Quoc Anh 1985 *PhD Thesis* University of Kishinev

[5] Ignatov A A and Romanov U A 1978 *Izv. Vyssh. Uch. Zaved. Radiofiz.* **21** 132

[6] Mensah S Y and Kangah G K 1992 *J. Phys.: Condens. Matter* **4** 919–22

[7] Shmelev G M, Chaikovskii I A and Mensah S Y 1989 *Fiz. Tekh. Poluprov.* **23** 712–5

[8] Epstein E M 1979 *Fiz. Tverd. Tela* **21** 242–4

[9] Milton A and Irene A S 1964 *Handbook of Mathematical Tables (Nat. Bur. Stand. Appl. Math. Ser. 55)* (Washington, DC: US Department of Commerce)

[10] Shik A Y 1975 *Sov. Phys.–Semicond.* **8** 159–209

[11] Kubakaddi S S, Mutimani B G and Jali V M 1986 *Phys. Status Solidi* b **137** 683